

Let us assume that $3 + 2\sqrt{5}$ is a rational number

So,

$$3 + 2\sqrt{5} = \frac{a}{b} \quad (\text{where, } a \text{ and } b \text{ are co-primes})$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a-3b}{b}$$

$$\sqrt{5} = \left(\frac{a-3b}{b} \right) \times \frac{1}{2}$$

$$\sqrt{5} = \frac{a-3b}{2b}$$

Here a and b are integers, we get $\frac{a-3b}{2b}$ as rational number and $\sqrt{5}$ is an irrational.

Since rational \neq irrational.

So there is a contradiction, which has arisen due to false assumption.

Hence $3 + 2\sqrt{5}$ is an irrational number.